Corrections Savoir Fi.2

Corrigé Exercice 6

1) $\int_0^1 f(t)dt = G(1) - G(0) = \left((3-1)e^1\right) - \left((0-1)e^0\right) = \mathbf{2e+1}$

2) **a.** $F'(x) = 2x - 2\times\frac{1}{x} = \frac{2x^2-2}{x} = \frac{2(x^2-1)}{x} = f(x)$ donc $F' = f$ et F est bien une primitive de f

b. $\int_1^e f(t)dt = F(e) - F(1) = (e^2 - 2\ln(e)) - (1 - 2\ln(1)) = e^2 - 2 - 1 + 0 = \mathbf{e^2 - 3}$

Corrigé Exercice 7

1) $\mathcal{A} = \int_1^2 \left(2 - \frac{4}{x}\right) dx = [2x - 4\ln x]_1^2$
$= (2\times 2 - 4\ln 2) - (2\times 1 - 4\ln 1)$
$= 4 - 4\ln 2 - 2 + 0 = \mathbf{2 - 4\ln 2}$

$\mathcal{B} = \int_0^1 (2t^2-1)\,dt = \left[\frac{2}{3}t^3 - t\right]_0^1$
$= \left(\frac{2}{3}\times 1^3 - 1\right) - \left(\frac{2}{3}\times 0^3 - 0\right)$
$= \frac{2}{3} - 1 = -\mathbf{\frac{1}{3}}$

$\mathcal{C} = \int_0^4 4e^{1-\frac{x}{2}}\,du = \left[-8e^{1-\frac{x}{2}}\right]_0^4$
$= \left(-8e^{1-\frac{4}{2}}\right) - \left(-8e^{1-0}\right)$
$= -8e^{-1} + 8e = \mathbf{8\left(e - \frac{1}{e}\right)}$

$\mathcal{D} = \int_0^3 e^{3\lambda}\,d\lambda = \left[\frac{1}{3}e^{3\lambda}\right]_0^3$
$= \left(\frac{1}{3}e^{3\times 3}\right) - \left(\frac{1}{3}e^{3\times 0}\right)$
$= \frac{1}{3}e^9 - \frac{1}{3} = \mathbf{\frac{1}{3}(e^9 - 1)}$

$\mathcal{E} = \int_0^{e-1} \frac{1}{x+1}\,dx = [\ln(x+1)]_0^{e-1}$
$= (\ln(e-1+1)) - (\ln(0+1))$
$= \ln e - \ln 1 = \mathbf{1}$

2) $I = \int_e^{e^2} \frac{1}{x\ln x}\,dx = [\ln(\ln x)]_e^{e^2}$
$= (\ln(\ln(e^2))) - (\ln(\ln e))$
$= \ln(2\ln e) - \ln(1) = \mathbf{\ln 2}$

Corrigé Exercice 8

$$on\ a\quad 3 - \frac{3}{1+e^{-2x}} = \frac{3(1+e^{-2x}) - 3}{1+e^{-2x}} = \frac{3e^{-2x}}{1+e^{-2x}}$$

$$\int_0^a \left(3 - \frac{3}{1+e^{-2x}}\right) dx = \int_0^a \frac{3e^{-2x}}{1+e^{-2x}}\,dx = \left[\frac{3}{-2}\ln(1+e^{-2x})\right]_0^a$$

$$= \left(-\frac{3}{2}\ln(1+e^{-2a})\right) - \frac{3}{2}\ln(1+e^0) = -\frac{3}{2}\ln(1+e^{-2a}) + \frac{3}{2}\ln 2 = \mathbf{\frac{3}{2}\ln\left(\frac{2}{1+e^{-2a}}\right)}$$

Corrigé Exercice 9

1) $F_1(x) = \int_1^x (1 - e^{2\lambda})\,d\lambda = \left[\lambda - \frac{1}{2}e^{2\lambda}\right]_1^x = \left(x - \frac{1}{2}e^{2x}\right) - \left(1 - \frac{1}{2}e^2\right) = \mathbf{x - \frac{1}{2}e^{2x} - 1 + \frac{1}{2}e^2}$

$$F_2(x) = \int_e^x \frac{2}{t}\,dt = [2\ln t]_e^x = (2\ln x) - (2\ln e) = \mathbf{2(\ln x - 1)}$$

$$F_3(x) = \int_x^1 (2a-3)da = [a^2 - 3a]_x^1 = (1^2 - 3\times 1) - (x^2 - 3x) = \mathbf{-x^2 + 3x - 2}$$

2) ***a.*** $$a + \frac{b}{x+3} = \frac{ax + 3a + b}{x+3} = \frac{1-x}{x+3} \Rightarrow \begin{cases} a = -1 \\ 3a + b = 1 \end{cases} \Leftrightarrow \begin{cases} a = -1 \\ b = 4 \end{cases}$$

b. $$G(T) = \int_0^T \frac{(1-x)}{x+3}dx = \int_0^T \left(-1 + \frac{4}{x+3}\right)dx = [-x + 4\ln(x+3)]_0^T$$

$$= (-T + 4\ln(T+3)) - (0 + 4\ln(3)) = \mathbf{-T + 4\ln\left(\frac{T+3}{3}\right)}$$

c. $$\int_0^1 \frac{(1-x)}{x+3}dx = G(1) = -1 + 4\ln\left(\frac{1+3}{3}\right) = \mathbf{-1 + 4\ln\frac{4}{3}}$$