Corrigé Exercice 6

1) a) $\bar{z} = \mathbf{2 - 4i}$ **b)** $\bar{z} = \mathbf{-3 + 2i}$ **c)** $\bar{z} = -i + (2 + 3i) = \mathbf{2 + 2i}$ **d)** $\bar{z} = +3i + 2 = \mathbf{2 + 3i}$

2) a) $z.\bar{z} = 3^2 + 1^2 = \mathbf{10}$ **b)** $z.\bar{z} = 4^2 + (-1)^2 = \mathbf{17}$

c) $z.\bar{z} = \left(\sqrt{2}\right)^2 + (-1)^2 = 2 + 1 = \mathbf{3}$ **d)** $z.\bar{z} = 3^2 + \left(\sqrt{3}\right)^2 = \mathbf{12}$

3) $f(2 - 3i) = (2 - 3i)(2 + 3i + i) = (2 - 3i)(2 + 4i) = 4 + 8i - 6i - 12i^2 = \mathbf{16 + 2i}$

Corrigé Exercice 7

1) a) $z = \frac{1}{i} = \frac{1\times(-i)}{i\times -i} = -\frac{i}{1} = \boldsymbol{-i}$

b) $z = \frac{3+i}{1+2i} = \frac{(3+i)(1-2i)}{1^2+2^2} = \frac{3-6i+i+2}{5} = \frac{5-5i}{5} = \mathbf{1 - i}$

c) $z = \frac{1}{2-i\sqrt{3}} = \frac{1(2+i\sqrt{3})}{(2-i\sqrt{3})(2+i\sqrt{3})} = \frac{2+i\sqrt{3}}{4+\sqrt{3}^2} = \frac{2+i\sqrt{3}}{4+3} = \boldsymbol{\frac{2}{7} + i\frac{\sqrt{3}}{7}}$

d) $z = \frac{1-5i}{2i} = \frac{(1-5i)(-2i)}{2i(-2i)} = \frac{-2i+10i^2}{4} = -\frac{10}{4} - \frac{2}{4}i = \boldsymbol{-\frac{5}{2} - \frac{1}{2}i}$

e) $z = \frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{1-2i+i^2}{1+1} = \frac{-2i}{2} = \boldsymbol{-i}$

2) $f(1 + 3i) = \frac{1+i}{2(1+3i-i)} = \frac{1+i}{2(1+2i)} = \frac{(1+i)(1-2i)}{2(1+2i)(1-2i)} = \frac{1-2i+i-2i^2}{2(1+4)} = \frac{3-i}{10} = \boldsymbol{\frac{3}{10} - \frac{1}{10}i}$

En bonus : $\frac{1}{\bar{z}} = \frac{1}{a-ib} = \boldsymbol{\frac{a+ib}{a^2+b^2}}$ et $\frac{z}{z'} = \frac{a+ib}{a'+ib'} = \frac{(a+ib)(a'-ib')}{a'^2+b'^2} = \boldsymbol{\frac{aa'+bb'+i(a'b-ab')}{a'^2+b'^2}}$

Corrigé Exercice 8

a. $\overline{\mathcal{Z}_1} = \bar{z} + \bar{\bar{z}} = \bar{z} + z = \mathcal{Z}_1$ $\Rightarrow$ le nombre **$\mathcal{Z}_1$ est réel**

b. $\overline{\mathcal{Z}_2} = \bar{\bar{z}} - \bar{z} = z - \bar{z} = -(\bar{z} - z) = -\mathcal{Z}_2$ $\Rightarrow$ le nombre **$\mathcal{Z}_2$ est un imaginaire pur**

c. $\overline{\mathcal{Z}_3} = \frac{1}{\bar{\bar{z}}} + \frac{1}{\bar{z}} = \frac{1}{z} + \frac{1}{\bar{z}} = \mathcal{Z}_3$ $\Rightarrow$ **$\mathcal{Z}_3$ est réel**

$\overline{\mathcal{Z}_4} = \frac{1}{\bar{z}} - \frac{1}{\bar{\bar{z}}} = \frac{1}{\bar{z}} - \frac{1}{z} = -\left(\frac{1}{z} - \frac{1}{\bar{z}}\right) = -\mathcal{Z}_4$ $\Rightarrow$ **$\mathcal{Z}_4$ est un imaginaire pur.**